Midterm Exam Calculus 1

28 September 2015, 9.00-11.00.

Write on each page your name and student number, and on the first page your tutorial group. The use of annotations, books and calculators is not permitted. All answers must be supported by arguments/work. Success.

1. (a) Formulate the principle of mathematical induction.
   (b) Use the principle of mathematical induction to show that if $h > -1$ then for all natural numbers $n \geq 1$

$$(1+h)^n \geq 1 + nh$$

2. The function f is given by $f(x) = x \sin x$. The n-th derivative of f is denoted by $f^{(n)}(x)$. Prove that the $2n$-th derivative of f is given by

$$f^{(2n)}(x) \;=\; (-1)^n \left(x \sin\left(x\right) - 2n \cos\left(x\right)\right)$$

   for all positive integers $n \geq 1$

3. Solve

$$z^2 = 1 - i$$

   and sketch the solutions in the complex plane.

4. Determine all complex numbers z satisfying

$$\mathrm{e}^{2z} = \mathrm{e}^{-z}$$

5. Prove - using the precise ϵ,δ definition of a limit - that

$$\lim_{x \to 0^+} \sqrt{x} \;=\; 0$$

Maximum score:

1a	0.5	2:	1.8	3:	1.8	4:	1.8	5:	1.8
b	1.3								

Total: 9 +1 (free) = 10.